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Problem of the Separation of Powers in Public Law From the Perspective of Boolean Algebra of Natural Law
(Constitutional Law and Discrete Mathematical Model of the Philosophical and Legal Doctrines of Rousseau and Hegel)

Abstract. The article sets out to investigate the problem of contradiction inherent in the mutual restraint of divided powers in public law. In order to do so, the work considers the algebraic aspect of the problem. The main methods of research are discrete mathematical modeling and the hypothetico-deductive method. Scientific novelty of the research result: for the first time in the field of constitutional law, the necessity of a contradiction in authority over the people for the reliable provision of the indivisible sovereignty of the people – i.e. its exercise of power (democracy) – is substantiated on a *strictly deductive* basis (by careful calculation of the compositions of the corresponding value functions).

Keywords: principle of separation of powers; Rousseau; sovereignty; indivisibility sovereignty; sovereignty of the people; democracy; power over the people; Hegel; internal contradiction; algebra of natural law

Introduction to the problematic. In the theory of state law in the 19th century, there was (and still is) a seemingly intellectually respectable tendency to consider the doctrine of the separation of powers (Locke 1988: 346-356; Montesquieu 1999: 138-139; Rousseau 1998: 231, 254) to be theoretically erroneous and practically harmful. Representatives of this trend include Professor of Law Léon Duguit and Professor of Law Bohdan Kistiakowski. In their works (Duguit 1908: 430-458; Kistyakovsky 1999: 472-488) they argued that the separation of powers is impossible, since their inevitable *interaction, interconnection, mutual adaptation, mutual limitation, mutual restraint represent a contradiction,* and the internally contradictory is impossible. The phrase "internal contradiction is impossible" is involuntarily associated with formal logic and with G.V.F. Hegel, whose teaching, according to some, is incompatible with the formal logical teaching on contradiction. It seems that the problem emerging in this particular relation is that of *antinomy*. However, the present article demonstrates that this *only appears to be the case*. The *illusion* (apparency) that resembles the truth is dispelled with the help of the hypothetico-deductive method and the systematic use of the conceptual apparatus of discrete mathematics. While the internal contradiction of divided power over the people noted by Kistyakovsky does indeed exist, according to the results of the study presented below, it is not dangerous for the people, but on the contrary, is expedient for ensuring their sovereignty. The author of the famous "Science of Logic" (who proclaimed in it that *all phenomena are internally contradictory*), when discussing the separation of powers, wrote:

> "...We must mention the idea of the *necessary separation of powers* in the state – an extremely important definition, which, taken in its true sense, could rightfully be considered as a guarantee of public freedom; but precisely those who imagine that they speak of it with enthusiasm and love know nothing about it and do not want to know, for it is precisely in it that the moment of *rational certainty* lies. The principle of separation of powers also contains an essential element of difference, of real rationality; however, in the understanding of abstract reason, it contains partly a false definition of the absolute independence of powers in relation to each other, and partly a one-sided understanding of their relationship to each other as negative, as a mutual *limitation*. This view presupposes hostility, the fear of each of the powers of what the other carries out against it as evil, and at the same time the determination of opposition to it and the establishment by means of such a counterweight of a general equilibrium, but not of a living unity. [...].
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> The *independence* of powers, for example, *the executive* and *the legislative,* as they are usually called, directly presupposes, as we have seen on a large scale, the destruction of the

> state – or, since the state is essentially preserved, a struggle arises as a result of which one power subordinates another and thereby creates a unity, whatever its character, and thus saves the essential, the existence of the state" (Hegel 1990: 309-310).

From this quote it follows that not only Duguit and Kistyakovsky, but to some extent also Hegel, are among the authoritative critics of the system of separation of powers as a system of *reciprocal limitations* (checks and balances). Although this theoretical *problem-contradiction* is obvious, there has until now been no successful attempt to develop a formally logically consistent solution to it on the part of constitutional law theorists, whether in the Russian or foreign scholarly literature. Jurists typically avoid the indicated abstract-theoretical antinomy problem, creating instead in their interlocutors and in themselves a plausible *illusion* of its solution by mean of a formal-logically contradictory agglomeration of complex sentences of a *purely natural* language, which is known for the intractable *polysemy* of its words and phrases. Jurists all over the planet also apparently fail to take into account that the word "state" itself has *multiple meanings*, being used in natural human language to connote at least *two qualitatively different value-functional meanings* (and at most – even in four). What are these *qualitatively different meanings* (why are there exactly 4 of them, and not 3 or 5) and is it possible to *strictly formally* define them, as required by legal culture itself? For all professional jurists of the past, without exception, as well as for all contemporary professional jurists of planet Earth, even the question itself is incomprehensible. But this cannot go on forever: it is time already *to start acting decisively* in the direction indicated by O. Spengler *more than a hundred years ago*, namely, to begin the systematic use of *the artificial* language of discrete mathematics, which is necessary for the systematic study of value *functions* (Spengler 1928: 67, 82, 83) and their compositions in the already existing *algebraic system of natural law* (Lobovikov 2011; Lobovikov 2014; Lobovikov 2016; Lobovikov 2020; Lobovikov 2022; Lobovikov 2023). When referring to Spengler, we have in mind first of all the following statements:

> "Classical law is a law of bodies. In the general stock composing the world it distinguishes bodily Persons and bodily

Things and, like a sort of Euclidean mathematic of public life, establishes ratios between them. The affinity between mathematical and legal thought is very close" (Spengler 1928: 67).

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"It must be emphasized then – and with all rigor – that Classical law was a law of *bodies* while ours is a law of *functions*. The Romans created a juristic static; our task is juristic dynamics. For us persons are not bodies, but units of force and will; and things are not bodies, but aims, means and creations of these units. The Classical relation between bodies was positional, but the relation between forces is called action" (Spengler 1928: 82).

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"The future will be called upon to transpose our entire legal thought into alignment with our higher physics and mathematics. Our whole social, economic, and technical life is waiting to be understood, at long last, in this wise. We shall need a century and more of keenest and deepest thought to arrive at the goal. And the prerequisite is a wholly new kind of preparatory training in the jurist" (Spengler 1928: 83).

In these quotes, Spengler brilliantly prophesies about a hypothetical *qualitatively new paradigm* of legal activity *of the future* (both in the sphere of scientific and theoretical research of law and in the sphere of professional education of lawyers). Thus, it is a statement not about what already exists, but about what, in Spengler's opinion, would be good to be. The attitude of jurists to Spengler's prophecy is ambiguous: some ignore it, because they do not want to agree with it for fundamental ideological and methodological reasons, while others could agree with it in words (and some of them would even sincerely like to), but cannot participate in the implementation of Spengler's project for objective reasons, namely because the legal education they received does not give them the opportunity to fruitfully use artificial languages, abstract concepts and methods of modern discrete mathematics. However, the situation is not hopeless: the world is changing rapidly: computerisation, informatisation, "digitalisation", "artificial intelligence", etc. irresistibly influence the formation of modern standards of higher education in general and legal education in particular. Therefore, at the present time, whoever can take part in the implementation of the project under discussion should do so to the extent possible.

Research methods (*precise definitions* of the basic concepts *necessary* to obtain and substantiate the above-mentioned new scientific results)

«*Ius est ars boni et aequi*»
(Law is the art of what is good and just).
Quoted from (Dozhdev 2016: 61).

The famous Latin saying "*Ius est ars boni et aequi*" of the outstanding ancient Roman lawyer Celsus (Publius Juventus the Younger[1]) was significantly cited by another outstanding ancient Roman lawyer Ulpian in his definition of the concept of "natural law" that interests us in this article. Ulpian wrote:

> "When a man means to give his attention to law (jus), he ought first to know whence the term jus is derived. Now jus is so called from justitia; in fact, according to the nice definition of Celsus, jus is the art of what is good and fair. 1. Of this art we may deservedly be called the priests; we cherish justice and profess the knowledge of what is good and fair, we separate what is fair from what is unfair, we discriminate between what is allowed and what is forbidden, we desire to make men good, not only by putting them in fear of penalties, but also by appealing to them through rewards, proceeding, if I am not mistaken, on a real and not a pretended philosophy. 2. Of this subject there are two departments, public law and private law. Public law is that which regards the constitution of the Roman state, private law looks at the interest of individuals; as a matter of fact, some things are beneficial from the point of view of the state, and some with reference to private persons. Public

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[1] His father, who was also an ancient Roman jurist, went under the same name "Publius Juventius Celsus"; therefore, in order to distinguish the son from the father, the addition of the word *Younger* (or *son*) to his name is usually used. The full name of the author of the Latin saying Ius est ars boni et aequi is given as follows: *Celsus, Publius Iuventius Titus Aufidius Hoenius Severianus.*

law is concerned with sacred rites, with priests, with public officers. Private law has a threefold division – it is deduced partly from the rules of natural law, partly from those of the jus gentium, partly from those of the civil law. 3. Natural law is that which all animals have been taught by nature; this law is not peculiar to the human species, but is common to all animals which are produced on land or sea, and to fowls of the air as well. From it comes the union of man and woman called by us matrimony, and there with the procreation and rearing of children; we find in fact that animals in general, the very wild beasts, are marked by acquaintance with this law. 4. Jus gentium is the law used by the various tribes of mankind, and there is no difficulty in seeing that it falls short of natural law, as the latter is common to all animated beings, whereas the former is only common to human beings in respect of their mutual relations" (Monroe 1904: 3).

From the point of view of modern canons of scientific ethics, Ulpian's citation of Celsus's dictum was entirely correct: Ulpian clearly referred to Celsus when he stated that natural "law is the science of the good and just" (Peretersky 1984: 23; Monroe 1904: 3: Watson 1998: 1). Another outstanding ancient Roman lawyer – Paul (Paulus) also considered natural law to be a science of the *good* and just. He wrote: "The word 'right' is used in several senses: first, 'right' means that which is always just and good – such as natural law" (Peretersky 1984: 25; Watson 1998: 2-3). It is indisputable that good (benevolent) and evil (bad) are *evaluative* concepts; therefore, the consideration of natural law as a formal *axiology* (universal theory of moral and legal values) has deep foundations in the legal theory of the ancient Roman Empire. In turn, in his fascinatingly written philosophical article "Mathematics and Good" (Whitehead 1990), the outstanding twentieth-century mathematician drew special attention to the possibility of fruitfully applying abstract concepts and effective methods of mathematics (especially algebra) to significantly clarify the vague idea of natural law (the concept of good) and to transform it into a true scientific theory by means of precisely formulating its strictly formally defined universal laws in the artificial language of mathematics.

In order to investigate the possibilities of fruitful application of the conceptual apparatus *of discrete mathematics* (Yablonsky

1979) to *natural law* in general, as well as to study the possible consequences of applying *a discrete mathematical model of natural law as a formal-axiological system* to the public-law problem of separation of powers in particular, this article uses the hypothetico-deductive method, artificial languages, and discretemathematical modeling – namely, the construction of algebraic systems suitable for this purpose. *The deductive method* is used in the present article to deduce and analyse the logical consequences of the fundamental *hypothesis*, according to which, in essence, *natural law is a universal formal axiology*: either (1) *of rational* activity (both *human* and *Divine*), or (2) *of any activity* (including irrational) *of any possible forms of life*, as Ulpian asserted (Peretersky 1984: 23; Monroe 1904: 3; Watson 1998: 1). We will not deal with the exclusive choice between the disjuncts included in the previous complex sentence in this article, since they do not exclude each other and both are of scientific and theoretical interest. That non-trivial hypothesis, the derivation of logical consequences from which this article is devoted, represents a coherent system of logically interconnected, strictly formally defined abstract concepts *of natural public (constitutional) law*. Now let us move directly to the formulation of precise definitions of the concepts included in this system.

Let us begin with the fact that, *by definition*, which is the basis of the hypothesis under study, *a two-valued algebraic system of natural law as formal axiology* is a triple of sets <П, O, R>, in which the symbol П denotes a non-empty (even potentially infinite) set of all such and only such existing or non-existent actions and persons (whether individual or collective), which are either good or bad, from the point of view of a certain subject of assessment (appraiser) ∑ (whether individual or collective, natural or artificial). It is obvious that ∑ – *variable*: changing its values can lead to a change in the estimates of specific elements of the set П. However, if the value of the variable ∑ is completely defined (rigidly fixed), then the estimates of specific elements of the set П turn out to be completely defined. Elements of the set П – actions or persons (individuals) are called *formal-axiological objects* of the theory of natural law (regardless of their existence or non-existence). The symbols “g (good)” and “b (bad)” denote *the axiological (value) meanings* of the elements of set П.

In the triple of sets <П,O,R>, the symbol O denotes the set of all *n-ary algebraic operationsth* (or simply *operations*), defined on the set Π. The elements of the set O are called *formal-axiological* operations of the two-valued *algebra of natural law* (as a formal axiology). In a two-valued algebraic system <Π,O,R>, *n-ary algebraic operations* defined on the set Π are those *functions* (and only those functions) that put into a one-to-one correspondence with each ordered *n*-k of elements of theset Π some element of the set Π, called the result of applying the said *n*-ary algebraic operation to the said ordered *n*-k of elements of the set Π. In other words, the *n-ary (algebraic) operation defined on the set* Π is the n-place function: $\Pi^n \to \Pi$. Here the symbol → denotes a "mapping" of one set into another (in the proper mathematical meaning of the word "mapping".

Let us now determine (in the above-mentioned triple of sets) the value of symbol R. In this triple of sets the symbol R denotes the set of all *n-ary formal-axiological relations*, defined on the set Π. For example, the binary relation defined below "*formal-axiological equivalence* (of elements of the set Π)" belongs to the set R. Since all three sets Π, O, and R are not empty, then, according to the precise definition of the concept of "algebraic system" generally accepted in modern mathematics, the triple discussed in this paper <Π,O,R> represents *an algebraic system* in the *strictly mathematical* sense of the term (Maltsev 1970; Cohn 1968).

The results of natural-legal (i.e. moral-legal) algebraic operations defined on a set Π are elements of the set Π. Consequently, by the definition of the set Π, they are either good or bad (from the point of view of ∑). Between the axiological values and – g (good) or b (bad) – of those elements of the set Π to which the moral-legal algebraic operation defined on the set Π was applied, and axiologically the value (g or b) of the result of this operation, there exists a *value-functional* correlation. The value value (g or b) of the result of an algebraic operation defined on Π is the value of a certain *value function*, the permissible values of whose variables are the axiological values (g or b) of those elements of the set Π to which the aforementioned algebraic operation is applied.

By definition, a *value function in the broad sense of the term* is any and only such function for which the domain (of change) of values of this function is a two-element set of {g (good), b (bad)}. By definition, a *value function in the narrow sense* is such and only such a function whose domain of admissible values of its variables is the two-element set {g (good), b (bad)}, and the domain of (changes in) values of this function is the same two-element set. In other words, when discussing value functions *in the narrow sense of the term*, the following mappings are meant: {g,b} → {g,b}, if we are talking about functions determined by *one* value argument; {g,b}×{g,b}→ {g,b}, if we are talking about functions determined by *two* value arguments (here "×" denotes the Cartesian product of sets); $\{g,b\}^N$ → {g,b}, if we are talking about functions determined by *N*-value arguments, (here *N* denotes some finite positive integer). By definition, a *mixed value function* is any such and only such *value function* whose range of admissible values of variables is the set Π defined above. Thus, according to the definitions given above, the sets *of purely* value functions and *of mixed* value functions are not identical to each other, but they are both subsets of the set value functions in the broad sense of the term.

Now let us consider specific examples of elementary natural-legal algebraic operations (elements of the set O) and the corresponding moral-legal value functions. Let us begin by considering the moral and legal algebraic operationsdefined on Π, determined by *one* argument.

Glossary for the table below 1. Symbol *Дх* denotes a moral and legal act "*restraint, holding* (of what, whom) *x*". Symbol *Dx* – "*division, division, divisibility* (of what, whom) *x*". *Ex* – "*unity, indivisibility, inseparability, unification* (of what, whom) *x*". *Уx* – "*destruction, ruin, corruption* (of what, whom) *x*". *Sx* – "*salvation, preservation* (of what, whom) *x*". *Nx* – "*non-existence, absence* (of what, whom) *x*". *Бx* – "*being, existence* (of what, whom) *x*". *Lx* – "*freedom* (of what, whom, whose) *x*" or "*freedom for* (of what, whom) *x*". *Fx* – "*freedom from* (of what, whom) *x*". ®*x* – "*arbitrariness*, i.e. *absolute freedom* (of actions), *in relation to* (of what, whom) *x*". *ℬx* – "*freedom of moral choice* and the commission of an act, which can be constructed *from a pair* (acts) *x* and *Nx*". In other words, *ℬx* – "*freedom of moral choice* (actions) *in relation to* (what, to whom) *x*, or *non-existence of arbitrariness* (*absolute freedom* of actions) *in relation to* (what, to whom) *x*, i.e. *freedom from arbitrariness in relation to* (what, to whom) *x*". *Пx* – "*production, creation, creation* (of what, whom) *x*". In the two-valued algebra of natural law, the functional dependence of the moral and legal axiological (value) value (g – good, or b – bad) of each of the above-mentioned moral and legal acts on the moral and legal value value of the variable *x* is precisely determined by the following table 1.

When getting acquainted with the value tables presented here, it is necessary to keep in mind that the symbols *x, y, z*, denote some (any) elements of the set Π, in it, and are symbols having the form $\Omega^n x_1, \ldots, x_n$ и $\Psi^n y_1, \ldots, y_n$, denote such results of application of *n*-ary algebraic operations (belonging to the set O) to some (any) ordered *n*-k elements from Π, which (results) themselves are also elements of the set Π. It makes sense to recall that *axiological (value) values* (elements of the set Π) are called in the two-valued algebra of natural law the elements of the set {g (good), b (bad)}. The symbols √*x*, √*y*, √*z* denote *axiological values* of the variables *x, y, z* of the discussed *value functions* (determined by some positive integer number of value variables), while the symbols √*Дx*, √*Dx*, √$У^2xy$, √$Щ^2xy$ denote a*xiological values of value functions Дx, Dx,* $У^2xy$, $Щ^2xy$, respectively. Value functions √*x*, √*y*, √*z* take values from the set {g (good), b (bad)}. Axiological meanings of value functions √*Дx*, √*Dx*, √$У^2xy$, √$Щ^2xy$ also belong to the set {g (хорошо), b (плохо)}.

Table 1. *Unary* moral and legal operations

√x	√Дx	√Dx	√Ex	√Уx	√Sx	√Nx	√Бx	√Lx	√Fx	√®x	√ℬx	√Пx
g	b	b	g	b	g	b	g	g	b	b	g	g
b	g	g	b	g	b	g	b	b	g	b	g	b

In connection with the fact that in the philosophy of law the maxim “Law is the *mathematics of freedom*” is popular as a beautiful metaphor, it makes sense to pay special attention to the fact that, according to Table 1, in the *formal-axiological semantics* of natural language there are several *mathematically different value-functional meanings of the* word “freedom”. In the case of *two-valued* formal axiology there are exactly four of them, namely: √*Lx*, √*Fx*, √®*x*, √ℬ*x*. Previously, *these* four mathematically distinct *value-functional* meanings of the word “freedom” were tabulated and examined in an English-language article (Lobovikov 2022a).

Glossary for the following Table 2. Px – “*power (strength), dominance* (of what, whom) *x*”. *Ax* – “*anarchy* (what, whom, whose) *x*”. *Vx* – “*power (strength), dominance over* (what, whom) *x*”. *Wx* – “*anarchy over* (what, whom) *x*”. *Ox* – “*limitation, definition, i.e. establishment of limits for* (what, whom) *x*” or “*limitation, certainty* (of what, whom) *x*”. *Jx* – “*judgement* (what, whom, whose) *x*”. *Ux* – “*judgement over* (what, whom) *x*”. *3x* – “*law* (what, whom, whose) *x*”. *Zx* – “*law for* (what, whom) *x, as a means of regulation, ordering, regulation* (what, whom) *x*". *Иx* – "*execution, implementation, fulfillment, realisation* (of what) *x*". *Гx* – "*state* (what, whom, whose) *x*". *Rx* – "*state over* (what, by whom) *x*". The value values of the results of the unary algebraic operations (defined on the set П), listed above in this glossary are precisely defined by the value Table 2 below.

Table 2. *Unary* operations of natural law algebra

√*x*	√*Px*	√*Ax*	√*Vx*	√*Wx*	√*Ox*	√*Jx*	√*Ux*	√*Зx*	√*Zx*	√*Иx*	√*Гx*	√*Rx*
g	g	b	b	g	b	g	b	g	b	g	g	b
b	b	g	g	b	g	b	g	b	g	b	b	g

Glossary for the value table below Table 3. Hx – "*people* (what, whom, whose) *x*". *Яx* – "*sovereignty* (of what, whom, whose) *x*" or "*sovereign* (what, who) *x*". ∇*x* – "*sovereignty over* (of what, whom) *x*". *Ьx* – "*connectedness*, *constraint* (of what, whom) *x*". Symbol ↑*x* – "*subordination, obedience, submission, subservience* (to what, to whom) *x*". Symbol ↓*x* – "*subordination, obedience, submission, subservience* (of what, whom, whose) *x*". #*x* – "*the opposite of* (of what, whom, whose) *x*" or "*the opposite of* (of what, whom) *x*". ☼*x* – "*democracy* (of what, whom, whose) *x*". ♀*x* – "*democracy over* (what, whom) *x*". The axiological meanings of the results of these unary algebraic operations (defined on the set П) are precisely defined by Table 3 located below.

Table 3. *Unary* operations on elements of П

√*x*	√*Hx*	√*Яx*	√∇*x*	√*Ьx*	√↑*x*	√↓*x*	√#*x*	√☼*x*	√♀*x*
g	g	g	b	b	g	b	b	g	b
b	b	b	g	g	b	g	g	b	g

Now let us consider specific examples of elementary moral and legal algebraic operations (elements of set O) defined on the set П and the corresponding moral and legal value functions determined by two moral and legal value arguments.

Glossary for Table below 4. Let the symbol $У^2xy$ denote the binary moral and legal algebraic operation "*destruction, ruin, decay, corruption* (of what, of whom) *x* (of what, by whom) *y*". (Here and further in the text of this article, the upper numeric index 2 informs that the indexed symbol denotes binary operation of the algebra of natural law.) Symbol S^2xy denotes binary moral-legal algebraic operation "*salvation, preservation* (of what, whom) *x* (by what, by whom) *y*". K^2xy denotes the binary moral and legal algebraic operation "*unity, union* (of what, of whom) *x* with (of what, of whom) *y*". D^2xy – "*division, separateness* (of what, whom) *x* and (of what, whom) *y*". $Д^2xy$ – "*containment, holding, holding, holding* (of what, whom) *x* (of what, by whom) *y*". O^2xy – "*limitation, definition, i.e. establishment of limits* (by whom, what) *y for* (of what, whom) *x*". V^2xy – "*coercion or superiority in force* (of what, whom, whose) *y over* (of what, by whom) *x*". P^2xy – "*power* (of what, whom, whose) *y over* (of what, by whom) *x*". $Г^2xy$ – "*state* (of what, whom, whose) *y over* (by what, by whom) *x*". The functional dependence of the axiological values of the results of the above-mentioned binary operations of the algebra of natural law on the axiological values of the variables *x* and *y* is precisely determined by the following table 4.

Table 4. *Binary* operations algebra of natural law

$\surd x$	$\surd y$	$\surd У^2xy$	$\surd S^2xy$	$\surd K^2xy$	$\surd D^2xy$	$\surd Д^2xy$	$\surd O^2xy$	$\surd V^2xy$	$\surd P^2xy$	$\surd Г^2xy$
g	g	b	g	g	b	b	b	b	b	b
g	b	b	g	b	g	b	b	b	b	b
b	g	g	b	b	g	g	g	g	g	g
b	b	b	g	b	g	b	b	b	b	b

Glossary for the following value Table 5. J^2xy – "*judgement* (of what, whom, whose) *y over* (of what, by whom) *x*". Z^2xy – "*law* (of what, whom, whose) *y for* (of what, whom) *x, as a means of regulation, ordering, regulation* (of what, whom) *x* (by what, by whom) *y*". $П^2xy$ – "*production, creation, making, implementation, execution* (of what, whom) *x* (by whom, than) *y*". X^2xy – "*anarchy* (of what, whom, whose) *y over* (of what, by whom) *x*". C^2xy – "*existence, being, presence, presence* (of what, whom, whose) *y in* (by what, whom) *x*". T^2xy – "*identity, identification* (of what, whom) *x with* (of what, by whom) *y*". U^2xy – "*excluding moral and legal choice between* (what, whom) *x and* (what, whom) *y*", i.e. (1) realisation of good and abstinence from bad, if *x* and *y* have opposite moral and legal meanings; (2) the realisation of the better and abstinence from the less good, if both (both *x* and and *y*) have the moral-legal meaning of "good"; (3) the realisation of the less bad and abstinence from the worse, if both

(both *x* and and *y*) have the moral-legal meaning of "bad"; (4) the realisation of any one and only one arbitrarily chosen either *x* or *y*, if there is no quantitative difference in their identical moral-legal meaning. N^2xy – "*unification non-realisation* (of what, whom) *x and non-realisation* (of what, whom) *y*". A^2xy – "*non-exclusive moral and legal choice between* (what, whom) *x and* (what, whom) *y*". More precisely, A^2xy is: (1) *realisation of* K^2xy, i.e. "*realisation of* (of what, whom) *y together with the realisation of* (of what, whom) *x*", if both (*x* and *y*) have the moral value "good"; (2) *realisation of* U^2xy, if *x* and *y* have qualitatively different moral meanings; (3) *realisation* U^2xy, if both (*x* and *y*) have the moral meaning "bad". The axiological meanings of the results of these *binary* algebraic operations (defined on the set П) are precisely determined by Table 5 below.

Table 5. *Binary* moral and legal operations

$\sqrt{x}$	$\sqrt{y}$	$\sqrt{J^2xy}$	$\sqrt{Z^2xy}$	$\sqrt{П^2xy}$	$\sqrt{X^2xy}$	$\sqrt{C^2xy}$	$\sqrt{T^2xy}$	$\sqrt{U^2xy}$	$\sqrt{N^2xy}$	A^2xy
g	g	b	b	g	g	g	g	b	b	g
g	b	b	b	g	g	b	b	g	b	g
b	g	g	g	b	b	g	b	g	b	g
b	b	b	b	g	g	g	g	b	g	b

Glossary for Table 6. The symbol $Ь^2xy$ means "*connectedness, connection* (of what, whom) *x* (by what, by whom) *y*". $Ч^2xy$ – "*contradiction, opposition* (of what, whom) *y* (of what, whom) *x*". $Щ^2xy$ – "*transformation* (of what, whom) *x into* (what, whom) *y*". Y^2xy – "*impact, action y on x*". R^2xy – "*counteraction* (of what, whom) *y* (to what, whom) *x*".

Table 6. *Binary* operations

$\sqrt{x}$	$\sqrt{y}$	$\sqrt{Ь^2xy}$	$\sqrt{Ч^2xy}$	$\sqrt{Щ^2xy}$	$\sqrt{Y^2xy}$	$\sqrt{R^2xy}$
g	g	b	b	b	b	b
g	b	b	b	b	b	b
b	g	g	g	g	g	g
b	b	b	b	b	b	b

Now, from the above-presented precise tabular definitions of value values, defined on the set П of algebraic operations (belonging to the set O), we will move on to strictly formal definitions of the basic concepts of the *modern* (precisely formulated in artificial mathematical language) theory of natural law: "*the law of natural law*", "*formal-axiological contradiction*", "*formal-axiological equivalence of the elements of the set* Π".

DEFINITION DF-1. By definition, for any elements φ and λ of the set Π, which have, respectively, moral and legal forms $\varphi(x_1, \ldots, x_n)$ and $\lambda(y_1, \ldots, y_k)$, it is true that λ and φ *are formally-axiologically equivalent (in natural law),* if and only if, for any axiological values of the moral and legal forms $x_1, \ldots, x_n$ and $y_1, \ldots, y_k$, it is true that $\surd\varphi(\surd x_1, \ldots, \surd x_n) = \surd\lambda(\surd y_1, \ldots, \surd y_k)$.

DEFINITION DF-2. By definition, any element φ of the set Π, having moral-legal form $\varphi(x_1, \ldots, x_n)$, is a (necessarily universal and immutable) *natural law* or (which is the same) a *formal-axiological law*, if and only if $\surd\varphi(\surd x_1, \ldots, \surd x_n) = g$, for any value values of moral and legal forms $x_1, \ldots, x_n$. Let us agree to denote *formal-axiological law* with the symbol @.

DEFINITION DF-3. By definition, any element φ of the set Π, having the moral-legal form $\varphi(x_1, \ldots, x_n)$, is *a formal-axiological contradiction* or (which is the same) n*on-fulfillment (violation, crime) of the law of natural law*, if and only if $\surd\varphi(\surd x_1, \ldots, \surd x_n) = b$, for any value values of moral and legal forms $x_1, \ldots, x_n$. Let us agree to designate the f*ormal-axiological contradiction* with the symbol ©.

New scientific results (obtained using the above strictly defined abstract concepts and methods).

> "I will allow myself to doubt that between the two schools, the school of Montesquieu and the school of Rousseau, there really exists that deep line of difference which is usually drawn, and that both will not serve to form a single political doctrine..." (Kovalevsky 1895: 613).

If the reader has fully realised and is ready to systematically use in his reasoning about natural public law in general and about the problem of separation of powers in particular all those strictly formally defined above concepts of the two-valued algebraic system of jus-naturalism, he can independently double-check (by carefully "calculating" the corresponding functions) the following algebraic "equations" and the translations of these "equations" into natural human language placed to the right of them (immediately after the colon).

1) $K^2O^2PxPyO^2PyPx$=+=©: mutual *limitation (definition)* of the powers *x* and *y* there is a contradiction (Hegel 1990: 309).

2) $K^2Д^2PxPyД^2PyPx$=+=©: inmutual *restraint* of the authorities *x* and *y* there is a contradiction.

3) $K^2R^2PxPyR^2PyPx$=+=©: mutual *opposition* of the authorities *x* and *y* (i.e. their *action "in opposition"*, contrary to each other) is a contradiction (Hegel 1990: 309).

4) $K^2Y^2PxPyY^2PyPx$=+=©: the mutualaction of the authorities *x* and *y* there is a contradiction.

5) $K^2Ь^2PxPyЬ^2PyPx$=+=©: the relationship between the authorities of *x* and *y* is a contradiction.

6) $K^2Щ^2PxPyЩ^2PyPx$=+=©: mutual transformation of powers *x* and *y* there is a contradiction.

7) $Ч^2xx$ =+=©: self-contradiction *x* is a contradiction.

8) O^2xx=+=$Ч^2xx$: self-determination (self-limitation) *x* is a self-contradiction.

9) O^2xx=+=©: self-determination (self-limitation) *x* is a contradiction.

10) P^2xx=+=©: autocracy *x* is a contradiction.

11) C^2x©=+=*Nx*: being *contradictions* in *x* is equivalent to non-being (of what, whom) *x*.

12) C^2VHx©=+=*NVHx*: the existence of the *contradiction* within of power over the people *x* is equivalent to the non-existence of power over the people *x*.

13) *NVHx*=+=*EЯHx*=+=*ЯHx*: the non-existence of power over the people *x* is equivalent to the (indivisible) sovereignty of the people *x*. (Rousseau 1998: 209-211, 216-217, 220-224).

14) *Dx*=+=*Vx*: division (of what, whom) *x* is equivalent to power over *x*. (How can one not recall the famous credo of the politicians of Ancient Rome: "Divide and rule!").

15) *Dx*=+=*Nx*: division (of what, whom) *x* is equivalent to non-existence *x*.

16) *DPRHx*=+=*NPRHx*: division of state power over the people *x* is equivalent to non-existence of state power over the people *x*. (Rousseau 1998: 54, 209-211, 216-217, 220-224, 246).

17) *DPRHx*=+=*ЯHx*=+=*EЯHx*: the division of power of the state over the people *x* is equivalent to the (indivisible) sovereignty of the people *x*. (Rousseau 1998: 209-217).

In discussing the separation of powers, Hegel wrote about the being of *self-determination of x within x* (i.e. within itself) the following: "Only *self-determination* of the concept within itself, and not any other goals and considerations of utility, represents the source of the absolute origin of the distinct powers, and only thanks to it is the state organisation within itself rational and a reflection of the eternal reason" (Hegel 1990: 309-310). The quoted idea, in my opinion, is quite adequately clarified and modeled by the following equations.

18) C^2VxO^2VxVx=+=*Яx*: being (*self-determination* (power over *x*)) in (power over *x*) means sovereignty *x*. This equation heuristically significantly *models* what was said in (Hegel 1990: 309).

19) $C^2RxO^2PRxPRx$=+=*Яx*: being *self-determination* (power (of the state over *x*)) in the state over *x* means sovereignty *x*. This equation is a heuristically significant *model* of what was said in (Hegel 1990: 309).

20) $C^2RHxO^2PRHxPRHx$=+=*Яx*=+=*EЯx*: being *self-determination* (power (of the state over the people *x*)) in the state over the people *x* is equivalent to the (indivisible) sovereignty of the people *x*.

This was to be substantiated by a careful calculation of the compositions of the corresponding value functions, according to the strict definitions of the concepts given in Section 2 of this article.

Duguit and Kistyakovsky considered Montesquieu's teaching on the separation of powers and Rousseau's teaching on both the indivisibility of sovereignty in general and the indivisibility of the sovereignty of the people in democracy in particular to be logically incompatible (mutually exclusive) (Kistyakovsky 1999: 484). Kistyakovsky criticised Maksim Kovalevsky's attempts to logically and consistently combine the mentioned teachings into some kind of unified political doctrine. According to Kistyakovski, these attempts, undertaken in the work "The Origin of Modern Democracy" (Kovalevsky 1895: 612-658), were not only *in fact* unsuccessful; they *could not* be crowned with success (Kistyakovsky 1999: 484). However, according to the system of equationspresented above, this opinion of Kistyakovsky is erroneous, since the indicated system of equations: (1) is logically consistent; (2) is a discrete mathematical model *of the unification* of the discussed theory of Montesquieu and the discussed theory of Rousseau. Moreover, the above system of equations is also a discrete mathematical model of a very non-trivial *formal-axiological interpretation* of Hegel's dialectical teaching on the denial of internally contradictory being. Expressed in *a purely natural* (very *polysemantic*) language *"objective dialectic"* (Hegel 2021), to many rationally thinking scientists *seems like "the ravings of a madman"*, a striking example of irrationalism, but, in my opinion, its psychologically unexpected *formal-axiological interpretation*, being adequately expressed in some completely unambiguous *artificial* language, *can be completely rational* and heuristically valuable for both philosophy and constitutional law.

"Paradoxes" of two-valued Boolean algebra of logic and two-valued Boolean algebra of natural law. According to modern philosophy of science, *the immediate* subject of any abstract theory is not sensually perceived, but abstract idealised objects; *the laws of abstract theory are necessarily universal relations between its abstract idealised objects*. Attempts to directly compare theoretical laws with empirical material lead to "paradoxes" (psychologically naturally arising illusions of the inadequacy) of the theory. This general position of modern philosophy of science is well illustrated by the "paradoxes of material implication" in two-valued Boolean algebra of logic, but the abstract-theoretical laws of classical algebra of logic, relating to disjunction and conjunction, from the point of view of everyday consciousness, also seem clearly paradoxical[2]. The two-valued algebra of logic and the two-valued algebra of natural law are qualitatively different interpretations of the same algebraic structure – the two-valued Boolean algebra. Since between these two *qualitatively different interpretations* (areas of application) of the same *proper (purely) mathematical* apparatus there is a relationship of mutual *analogy*, it is entirely plausible that in

[2] For example, the conjunction expressed in natural language by the union "and" is *commutative*. Direct application of this law of logical theory to real life gives the following result: the statement "Jane gave birth and got married" is logically equivalent to the statement "Jane got married and gave birth". However, if the logical equivalence of these two statements were put to a vote by the broad masses of the people, then with a very high probability one can predict that it would be rejected by a majority of votes as clearly contradicting the rich empirical material from the personal lives of voters, thus representing a "paradox"!

the Boolean algebra of natural law under discussion, the ordinary consciousness of a normal person also cannot fail to notice a certain strangeness of its abstract idealised objects and the paradoxicality of its laws representing the necessarily universal relations between its abstract idealised objects.

Let us consider this using a specific example of the above-defined operations $\uparrow x$ – "subordination, obedience, submission, subservience (to what, to whom) x" and $\downarrow x$ – "subordination, obedience, submission, subservience (of what, of whom) x". The reader may find that some of the meanings of the value functions introduced by the author look rather strange, especially if the reader is a strictly humanities scholar. Thus, for example, from Table 3 presented above, the reader learns that submission to "good x" is a good thing, and he probably will not argue with this. But why submission to someone on the part of this "good x" is assessed negatively may not be understood.

In order to eliminate such a psychologically naturally arising misunderstanding, it is advisable to once again draw the reader's attention to the fact that the *direct* objects of application of the theory of natural law are its abstract idealised objects, which are strictly formally defined in it. One such abstract idealised object is the "good subject (good person)". *According to the accepted definition* of a good subject, having freedom, he will do good by himself without subordination to anyone. Therefore, in the case under discussion, *submission, subjection to the good is excess*; a good person does not need to be subordinated: acting freely, he will do good by himself, and if he himself by himself does not do good without submission, then he is not good, but is bad (according to the accepted definition), and the subordination of the bad is good. Excess is a violation of measure, and this, according to Aristotle, is something bad (vice, evil). Therefore, being superfluous, the subordination of the good is bad, which is entirely consistent with the discussed tabular definition. Similar "paradoxes" also arise in everyday consciousness in connection with some other operations of the algebra of natural law. We will not consider all these "paradoxes" here now, since the volume of this article is limited, but we will note that all of them are successfully resolved by applying to them the precedent (by analogy) that was formulated above in connection with the algebraic operations$\uparrow x$ and $\downarrow x$.

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