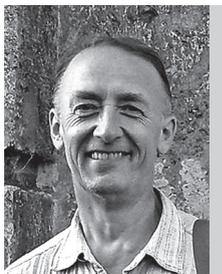


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## МОДЕЛИ ДЛЯ ФОРМАЛЬНОЙ АКСИОМАТИЧЕСКОЙ ТЕОРИИ ЗНАНИЯ $\Xi$



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### Аннотация

Определяется формальная аксиоматическая теория  $\Xi$ , представляющая собой философскую эпистемологию, и исследуется проблема ее логической непротиворечивости. Впервые выносятся на обсуждение такие качественно различные интерпретации аксиоматической системы  $\Xi$ , которые являются моделями для  $\Xi$ . С помощью этих моделей доказывается, что обсуждаемая формальная теория знания логически непротиворечива.

Ключевые понятия:

формальная-аксиоматическая-теория; эпистемология; интерпретация; модель; непротиворечивость.

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### 1. Introduction

A definition of the theory  $\Xi$  may be found in [15, 19–21]. During the oral presentation and discussion of  $\Xi$  at the World Congress on Universal Logic in Vichy, France, 2018, the logic consistency of  $\Xi$  was questioned. Moreover, some colleagues expressed the hypothesis that  $\Xi$  is inconsistent. Therefore, as in relation to philosophical epistemology,  $\Xi$  is a nontrivial novelty worthy of further development and systematical investigation, I have studied the consistency problem and submit results of the study below in this paper.

### 2. Definition of $\Xi$

For constructing a rigorous proof of logic consistency of the formal axiomatic epistemology theory  $\Xi$  it is indispensable to have a precise definition of that theory.

Therefore, the present paragraph 2 of this paper is aimed at making the reader acquainted with the rigorous formulation of  $\Xi$  which can be found, for instance, in [19–21]. According to the definition given in these papers, the logically formalized axiomatic epistemology system  $\Xi$  contains all symbols, expressions, formulae, axioms, and inference-rules of the classical propositional logic. Symbols  $q, p, d, \dots$  (called propositional letters) are *elementary* formulae of  $\Xi$ . Symbols  $\alpha, \beta, \omega, \pi, \dots$  (belonging to meta-language) stand for any formulae of  $\Xi$ . In general, the notion “formulae of  $\Xi$ ” is defined as follows.

- 1) All propositional letters  $q, p, d, \dots$  are formulae of  $\Xi$ .
- 2) If  $\alpha$  and  $\beta$  are formulae of  $\Xi$ , then all such expressions of the object-language of  $\Xi$ , which possess logic forms  $\neg\alpha, (\alpha \rightarrow \beta), (\alpha \leftrightarrow \beta), (\alpha \& \beta), (\alpha \vee \beta)$ , are formulae of  $\Xi$  as well.
- 3) If  $\alpha$  is a formula of  $\Xi$ , then  $\Psi\alpha$  is a formula of  $\Xi$  as well.
- 4) Successions of symbols (belonging to the alphabet of the object-language of  $\Xi$ ) are formulae of  $\Xi$ , only if this is so owing to the above-given items (1) – (3) of the present definition.

The symbol  $\Psi$  belonging to meta-language stands for any element of the set of modalities  $\{\square, \mathbf{K}, \mathbf{A}, \mathbf{E}, \mathbf{S}, \mathbf{T}, \mathbf{F}, \mathbf{P}, \mathbf{Z}, \mathbf{G}, \mathbf{O}, \mathbf{B}, \mathbf{U}, \mathbf{Y}\}$ . Symbol  $\square$  stands for the alethic modality “necessary”. Symbols  $\mathbf{K}, \mathbf{A}, \mathbf{E}, \mathbf{S}, \mathbf{T}, \mathbf{F}, \mathbf{P}, \mathbf{Z}$ , respectively, stand for modalities “agent knows that...”, “agent *a-priori* knows that...”, “agent *a-posteriori* knows that...”, “under some conditions in some space-and-time a person (immediately or by means of some tools) *sensually perceives* (has *sensual verification*) that...”, “it is *true* that...”, “agent *believes* that...”, “it is *provable* that...”, “there is an *algorithm* (a machine could be constructed) *for deciding* that...”.

Symbols  $\mathbf{G}, \mathbf{O}, \mathbf{B}, \mathbf{U}, \mathbf{Y}$ , respectively, stand for modalities “it is (*morally*) *good* that...”, “it is *obligatory* that...”, “it is *beautiful* that...”, “it is *useful* that...”, “it is *pleasant* that...”. Meanings of the mentioned symbols are defined by the following schemes of own-axioms of epistemology system  $\Xi$  which axioms are added to the axioms of classical propositional logic. Schemes of axioms and inference rules of the classical propositional logic are applicable to all formulae of  $\Xi$  (including the additional ones).

Axiom scheme AX-1:  $\mathbf{A}\alpha \rightarrow (\square\beta \rightarrow \beta)$ .

Axiom scheme AX-2:  $\mathbf{A}\alpha \rightarrow (\square(\alpha \rightarrow \beta) \rightarrow (\square\alpha \rightarrow \square\beta))$ .

Axiom scheme AX-3:  $\mathbf{A}\alpha \leftrightarrow (\mathbf{K}\alpha \& (\square\alpha \& \square\neg\mathbf{S}\alpha \& \square(\beta \leftrightarrow \Omega\beta)))$ .

Axiom scheme AX-4:  $\mathbf{E}\alpha \leftrightarrow (\mathbf{K}\alpha \& (\neg\square\alpha \vee \neg\square\neg\mathbf{S}\alpha \vee \neg\square(\beta \leftrightarrow \Omega\beta)))$ .

In AX-3 and AX-4, the symbol  $\Omega$  (belonging to the meta-language) stands for any element of the set  $\mathbf{R} = \{\square, \mathbf{K}, \mathbf{T}, \mathbf{F}, \mathbf{P}, \mathbf{Z}, \mathbf{G}, \mathbf{O}, \mathbf{B}, \mathbf{U}, \mathbf{Y}\}$ . Let elements of  $\mathbf{R}$  be called “*perfection-modalities*” or simply “*perfections*”.

### 3. Models of/for $\Xi$

Above the axioms of  $\Xi$  were defined by the axiom-schemes. Now first of all it is relevant to depart from the meta-language to the object-language, i. e. to move from the above axiom-schemes to the following axioms, respectively.

Axiom AX-1\*:  $\mathbf{A}q \rightarrow (\square p \rightarrow p)$ .

Axiom AX-2\*:  $\mathbf{A}q \rightarrow (\square(q \rightarrow p) \rightarrow (\square q \rightarrow \square p))$ .

Axiom AX-3\*:  $\mathbf{A}q \leftrightarrow (\mathbf{K}q \& (\square q \& \square\neg\mathbf{S}q \& \square(p \leftrightarrow \square p)))$ .

Axiom AX-4\*:  $\mathbf{Eq} \leftrightarrow (\mathbf{Kq} \ \& \ (\neg\Box\mathbf{q} \vee \neg\Box\neg\mathbf{Sq} \vee \neg\Box(\mathbf{p} \leftrightarrow \Box\mathbf{p})))$ .

These axioms are obtained from the corresponding axiom-schemes by substituting: propositional letter  $\mathbf{q}$  for  $\alpha$ ; propositional letter  $\mathbf{p}$  for  $\beta$ ;  $\Box$  for  $\Omega$ . In this paper such interpretations of/for  $\Xi$  are considered in which all the axioms of  $\Xi$  are true. Now everything is prepared for defining and discussing interpretation-functions to be used for the demonstration of consistency.

Let  $\oplus$  stand for an element of the set of classical binary connectives  $\{\rightarrow, \leftrightarrow, \&, \vee\}$ . Let  $@$  stand for an element of the set of below-considered interpretation-functions  $\{\forall, \nabla, \epsilon, \text{f}\}$ . It is a *common* aspect of the below-given definitions of the interpretation-functions under consideration in this paper that, for any  $@$ ,  $\oplus$ ,  $\omega$ , and  $\pi$ , it is true that:

- 1)  $@\neg\omega = \neg@ \omega$ ;
- 2)  $@(\omega \oplus \pi) = (@\omega \oplus @\pi)$ .

Now let us move to *specific* aspects of the interpretation-function-definitions under review in this paper.

### 3.1. Interpretation $\forall$

- 3)  $\forall\mathbf{q} = \text{true}$ .
- 4)  $\forall\mathbf{p} = \text{true}$ .
- 5)  $\forall\mathbf{Aq} = \text{true}$ .
- 6)  $\forall\mathbf{Kq} = \text{true}$ .
- 7)  $\forall\mathbf{Eq} = \text{false}$ .
- 8)  $\forall\mathbf{Sq} = \text{false}$ .

9) For any  $\omega$ ,  $\forall\Box\omega = \text{true}$ : *everything is necessary*; this is an expression of such an extremely rationalistic a-priori-ism philosophy which can be extracted from writings of Spinoza [28] and Leibniz [11–14].

In the interpretation  $\forall$ , all the axioms of  $\Xi$  are true, consequently,  $\Xi$  has a model, hence  $\Xi$  is consistent.

### 3.2. Interpretation $\nabla$

- 3)  $\nabla\mathbf{q} = \text{true}$ .
- 4)  $\nabla\mathbf{p} = \text{true}$ .
- 5)  $\nabla\mathbf{Aq} = \text{false}$ .
- 6)  $\nabla\mathbf{Kq} = \text{true}$ .
- 7)  $\nabla\mathbf{Eq} = \text{true}$ .
- 8)  $\nabla\mathbf{Sq} = \text{true}$ .

9) For any  $\omega$ ,  $\nabla\Box\omega = \text{false}$ : *nothing is necessary*; this is an expression of such an extreme sensualism-and-empiricism philosophy which can be extracted from writings of Locke [22], Hume [7, 8], Berkeley [5], Mach [23, 24], Popper [26, 27], and Wittgenstein [29].

In the interpretation  $\nabla$  all the axioms of  $\Xi$  are true, consequently,  $\Xi$  has a model, hence  $\Xi$  is consistent.

### 3.3. Interpretation $\epsilon$

- 3)  $\epsilon\mathbf{q} = \text{true}$ .

- 4)  $\epsilon p = \text{true}$ .
- 5)  $\epsilon Aq = \epsilon q$ .
- 6)  $\epsilon Kq = \epsilon q$ .
- 7)  $\epsilon Eq = \epsilon \neg q$ .
- 8)  $\epsilon Sq = \epsilon \neg q$ .
- 9) For any  $\omega$ ,  $\epsilon \Box \omega = \epsilon \omega$ .

In the interpretation  $\epsilon$ , all the axioms of  $\Xi$  are true, consequently,  $\Xi$  has a model, hence  $\Xi$  is consistent.

### 3.4. Interpretation $\xi$

- 3)  $\xi q = \text{true}$ .
- 4)  $\xi p = \text{true}$ .
- 5)  $\xi Aq = \xi \neg q$ .
- 6)  $\xi Kq = \xi q$ .
- 7)  $\xi Eq = \xi q$ .
- 8)  $\xi Sq = \xi q$ .
- 9) For any  $\omega$ ,  $\xi \Box \omega = \xi \neg \omega$ .

In the interpretation  $\xi$ , all the axioms of  $\Xi$  are true, consequently,  $\Xi$  has a model, hence  $\Xi$  is consistent.

## 4. Formal proofs of philosophically interesting theorems in $\Xi$

Strictly speaking, here I mean not proofs of theorems but schemes of proofs of schemes of theorems. They are the following.

### 4.1. Theorem-scheme ( $A\alpha \rightarrow (O\alpha \leftrightarrow G\alpha)$ )

Its formal proof (or, strictly speaking, scheme of proofs) in  $\Xi$  is the following succession of formulae-schemes.

- 1)  $A\alpha \leftrightarrow (K\alpha \ \& \ (\Box\alpha \ \& \ \Box\neg S\alpha \ \& \ \Box(\beta \leftrightarrow \Omega\beta))$ : axiom scheme AX-3.
  - 2)  $A\alpha$ : assumption.
  - 3)  $K\alpha \ \& \ \Box\alpha \ \& \ \Box\neg\neg S\alpha \ \& \ \Box(\beta \leftrightarrow \Omega\beta)$ : from 1 and 2 by propositional logic.
  - 4)  $\Box(\beta \leftrightarrow \Omega\beta)$ : from 3 by the rule of  $\&$ -elimination.
  - 5)  $(\beta \leftrightarrow \Omega\beta)$ : from 4 by the (limited) rule of  $\Box$ -elimination.
  - 6)  $(\beta \leftrightarrow G\beta)$ : from 5 by substituting  $G$  for  $\Omega$ .
  - 7)  $(\beta \leftrightarrow O\beta)$ : from 5 by substituting  $O$  for  $\Omega$ .
  - 8)  $(O\beta \leftrightarrow \beta)$ : from 7 by commutativity of  $\leftrightarrow$ .
  - 9)  $(O\beta \leftrightarrow G\beta)$ : from 8 and 6 by transitivity of  $\leftrightarrow$ .
  - 10)  $A\alpha \mid - (O\beta \leftrightarrow G\beta)$ : by 1–9.
  - 11)  $A\alpha \mid - (O\alpha \leftrightarrow G\alpha)$ : from 10 by substituting  $\alpha$  for  $\beta$ .
  - 12)  $\mid - (A\alpha \rightarrow (O\alpha \leftrightarrow G\alpha))$ : from 11 by the rule of introduction of  $\rightarrow$ .
- Here you are.

### 4.2. Theorem-scheme ( $A\alpha \rightarrow (O\alpha \leftrightarrow \Box\alpha)$ )

Its formal-proof-scheme is the following succession.

- 1)  $A\alpha \leftrightarrow (K\alpha \& (\Box\alpha \& \Box\neg Sa \& \Box(\beta \leftrightarrow \Omega\beta)))$ : axiom scheme AX-3.
- 2)  $A\alpha$ : assumption.
- 3)  $K\alpha \& \Box\alpha \& \Box\neg Sa \& \Box(\beta \leftrightarrow \Omega\beta)$ : from 1 and 2 by propositional logic.
- 4)  $\Box(\beta \leftrightarrow \Omega\beta)$ : from 3 by the rule of  $\&$ -elimination.
- 5)  $(\beta \leftrightarrow \Omega\beta)$ : from 4 by the (limited) rule of  $\Box$ -elimination.
- 6)  $(\beta \leftrightarrow \Box\beta)$ : from 5 by substituting  $\Box$  for  $\Omega$ .
- 7)  $(\beta \leftrightarrow O\beta)$ : from 5 by substituting  $O$  for  $\Omega$ .
- 8)  $(O\beta \leftrightarrow \beta)$ : from 7 by commutativity of  $\leftrightarrow$ .
- 9)  $(O\beta \leftrightarrow \Box\beta)$ : from 8 and 6 by transitivity of  $\leftrightarrow$ .
- 10)  $A\alpha \mid - (O\beta \leftrightarrow \Box\beta)$ : by 1–9.
- 11)  $A\alpha \mid - (O\alpha \leftrightarrow \Box\alpha)$ : from 10 by substituting  $\alpha$  for  $\beta$ .
- 12)  $\mid - (A\alpha \rightarrow (O\alpha \leftrightarrow \Box\alpha))$ : from 11 by the rule of introduction of  $\rightarrow$ .

Here you are.

Obviously, the above-given schemes of proofs are analogous; they are generalized by the following scheme of proofs of scheme of theorems in  $\Xi$ .

### 4.3. Theorem-scheme ( $A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha)$ )

For any  $\Sigma$  and  $\Omega$ , it is provable in  $\Xi$  that  $(A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha))$ , where the symbols  $\Sigma$  and  $\Omega$  (belonging to the meta-language) stand for any elements of the set  $R = \{\Box, K, T, F, P, Z, G, O, B, U, Y\}$ . (Elements of  $R$  are called *perfection-modalities*.) The following succession of schemes of formulae is a scheme of proofs of/for  $(A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha))$  in  $\Xi$ .

- 1)  $A\alpha \leftrightarrow (K\alpha \& (\Box\alpha \& \Box\neg Sa \& \Box(\beta \leftrightarrow \Omega\beta)))$ : axiom scheme AX-3.
- 2)  $A\alpha \rightarrow (K\alpha \& (\Box\alpha \& \Box\neg Sa \& \Box(\beta \leftrightarrow \Omega\beta)))$ : from 1 by the rule of elimination of  $\leftrightarrow$ .
- 3)  $A\alpha$ : assumption.
- 4)  $(K\alpha \& (\Box\alpha \& \Box\neg Sa \& \Box(\beta \leftrightarrow \Omega\beta)))$ : from 2 and 3 by *modus ponens*.
- 5)  $\Box(\beta \leftrightarrow \Omega\beta)$ : from 4 by the rule of elimination of  $\&$ .
- 6)  $(\beta \leftrightarrow \Omega\beta)$ : from 5 by the rule of elimination of  $\Box$ .
- 7)  $(\alpha \leftrightarrow \Sigma\alpha)$ : from 6 by substituting  $(\alpha$  for  $\beta$ , and  $\Sigma$  for  $\Omega)$ .
- 8)  $(\alpha \leftrightarrow \Omega\alpha)$ : from 6 by substituting  $(\alpha$  for  $\beta)$ .
- 9)  $(\Sigma\alpha \leftrightarrow \alpha)$ : from 7 by commutativity of  $\leftrightarrow$ .
- 10)  $(\Sigma\alpha \leftrightarrow \Omega\alpha)$ : from 9 and 8 by transitivity of  $\leftrightarrow$ .
- 11)  $A\alpha \mid - (\Sigma\alpha \leftrightarrow \Omega\alpha)$ : by 1–10.
- 12)  $\mid - A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha)$ : from 11 by the rule of introduction of  $\rightarrow$ .

From the viewpoint of purely mathematical technique, the proof of  $(A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha))$  is not interesting (too simple). But from the viewpoint of proper philosophy contents, the statement  $(A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha))$  is very interesting and important. Various concrete philosophical interpretations (particular cases) of that statement are well-known as fundamental philosophical principles of the rationalism (a-priori-ism). For example, the following specific philosophical interpretations of the theorem-scheme  $(A\alpha \rightarrow (\Sigma\alpha \leftrightarrow \Omega\alpha))$  are worth mentioning.

- a)  $A\alpha \rightarrow (G\alpha \leftrightarrow T\alpha)$ : the rationalistic principle of *optimism in ethics* by N. Malebranche and G. W. Leibniz.

- b)  $Aa \rightarrow (Ta \leftrightarrow Pa)$ : the rationalistic principle of *optimism in epistemology* by G.W. Leibniz and D. Hilbert. About modeling this principle see [15; 16; 19].
- c)  $Aa \rightarrow (Pa \leftrightarrow Za)$ : the rationalistic principle of *mechanistic (algorithmic) optimism in epistemology* by R. Lull (Lullus), G. W. Leibniz, and A.A. Lovelace (Augusta Ada King-Noel, Countess of Lovelace).
- d)  $Aa \rightarrow (\Box a \leftrightarrow Ga)$ : the rationalistic principle of equivalence between necessary being and (universal) goodness. This principle was expressed by some outstanding creators of Ancient-Roman-Law, for example, Ulpian, and some great theologians, for example, St. Tomas Aquinas [1; 2].
- e)  $Ap \rightarrow (Gp \leftrightarrow Bp)$ : the principle of *kalokagathia* (Socrates, Xenophon, Plato, Aristotle [2; 3]);
- f)  $Ap \rightarrow (Gp \leftrightarrow Up)$ : the principle of *utilitarianism* ethics (J. Bentham, J.-St. Mill [25]). About modeling this principle in  $\Xi$ , see [17; 19].
- g)  $Ap \rightarrow (Gp \leftrightarrow Yp)$ : the principle of *hedonism* ethics (Aristippus, Epicurus). Modeling this principle in  $\Xi$  is discussed in [17; 19].
- h)  $Ap \rightarrow (Bp \leftrightarrow Yp)$ : the principle of *hedonism in aesthetics*;
- i)  $Ap \rightarrow (Bp \leftrightarrow Up)$ : the principle of *beauty of useful* (and *usefulness of beauty*).
- j)  $Ap \rightarrow (Tp \leftrightarrow Up)$ : the principle of *pragmatism* in theory of truth (J. Dewey [6], W. James [9; 10], C.S. Peirce).
- k)  $Ap \rightarrow (Tp \leftrightarrow Bp)$ : the principle of *beauty as criterion of truth*. (W. Blake, P.A.M. Dirac).
- l)  $Ap \rightarrow (Pp \leftrightarrow Bp)$ : the principle of *beauty as criterion of proof* (S.S. Averincev).

#### 4.4. Theorem-scheme ( $Aa \rightarrow (\Box a \leftrightarrow \Box \Omega a)$ )

In addition to the above-said it is worth mentioning that the following succession of formula-schemes is a scheme of proofs (in  $\Xi$ ) of the philosophically interesting theorem-scheme ( $Aa \rightarrow (\Box a \leftrightarrow \Box \Omega a)$ ), where  $\Omega$  takes values from the set  $\mathbf{R}$ .

- 1)  $Aa \leftrightarrow (Ka \ \& \ (\Box a \ \& \ \Box \neg Sa \ \& \ \Box(\beta \leftrightarrow \Omega\beta))$ : axiom scheme AX-3.
- 2)  $Aa$ : assumption.
- 3)  $Ka \ \& \ \Box a \ \& \ \Box \neg Sa \ \& \ \Box(\beta \leftrightarrow \Omega\beta)$ : from 1 and 2 by propositional logic.
- 4)  $\Box(\beta \leftrightarrow \Omega\beta)$ : from 3 by the rule of  $\&$ -elimination.
- 5)  $\Box(a \leftrightarrow \Omega a)$ : from 4 by substituting  $a$  for  $\beta$ .
- 6)  $Aa \rightarrow (\Box(a \leftrightarrow \beta) \rightarrow (\Box a \leftrightarrow \Box\beta))$ : theorem scheme.
- 7)  $Aa \rightarrow (\Box(a \leftrightarrow \Omega a) \rightarrow (\Box a \leftrightarrow \Box\Omega a))$ : from 6 by substituting  $\Omega a$  for  $\beta$ .
- 8)  $\Box(a \leftrightarrow \Omega a) \rightarrow (\Box a \leftrightarrow \Box\Omega a)$ : from 7 and 2 by modus ponens.
- 9)  $(\Box a \leftrightarrow \Box\Omega a)$ : from 8 and 5 by modus ponens.
- 10)  $\Box(Aa \rightarrow (\Box a \leftrightarrow \Box\Omega a))$ : by the rule of introduction of  $\rightarrow$ .

Here you are.

The theorem-scheme ( $Aa \rightarrow (\Box a \leftrightarrow \Box \Omega a)$ ) may be instantiated by the following nontrivial philosophical principles.

- a)  $Aa \rightarrow (\Box a \leftrightarrow \Box Ga)$ : the natural-law principle of *equivalence of necessary being and necessary positive-moral-value (necessary goodness)*, represented in works of Aristotle, Ulpian, and Aquinas. About this see [18; 19].

b)  $\mathbf{Ap} \rightarrow (\Box p \leftrightarrow \Box \mathbf{Op})$ : the natural-law principle of *equivalence of necessary being and necessary norm (duty)*, represented in works of Cicero, I. Kant, and H. Kelsen. Of this principle see [18; 19].

From a) and b) it follows logically that  $\mathbf{Ap} \rightarrow (\Box \mathbf{Op} \leftrightarrow \Box \mathbf{Ga})$ : the principle of equivalence of the normative (deontic) and the evaluative options of formulating the natural-law doctrine [18; 19].

Gödel's necessitation rule does not belong to the set of inference rules of  $\Xi$ . Nevertheless, it is easy to demonstrate in  $\Xi$  that *under the condition* that  $\mathbf{Aa}$  (but *not in general*), the following (limited) inference-rule of *necessitation* is valid: "If  $\mathbf{Aa} \mid \neg \beta$ , then  $\mathbf{Aa} \mid \neg \Box \beta$ ". The following inference is a demonstration of this rule.

1.  $\mathbf{Aa} \leftrightarrow (\mathbf{Ka} \ \& \ (\Box a \ \& \ \neg \Box \mathbf{Sa} \ \& \ \Box (\beta \leftrightarrow \mathbf{O}\beta))$ : axiom scheme AX-3.
2.  $\mathbf{Aa}$ : assumption.
3.  $\mathbf{Ka} \ \& \ \Box a \ \& \ \neg \Box \mathbf{Sa} \ \& \ \Box (\beta \leftrightarrow \mathbf{O}\beta)$ : from 1 and 2 by propositional logic.
4.  $\Box (\beta \leftrightarrow \mathbf{O}\beta)$ : from 3 by the rule of &-elimination.
5.  $(\beta \leftrightarrow \mathbf{O}\beta)$ : from 4 by the (limited) rule of  $\Box$ -elimination.
6.  $\mathbf{Aa} \mid \neg (\beta \leftrightarrow \mathbf{O}\beta)$ : by 1–5.
7.  $\mathbf{Aa} \mid \neg (\beta \leftrightarrow \Box \beta)$ : from 6 by substituting  $\Box$  for  $\mathbf{O}$ .
8.  $\mathbf{Aa} \mid \neg \beta$ : is given.
9.  $\mathbf{Aa} \mid \neg \Box \beta$ : from 7 and 8 by propositional logic.
10. If  $\mathbf{Aa} \mid \neg \beta$  then  $\mathbf{Aa} \mid \neg \Box \beta$ : by 1–9.

## 5. Conclusion

As there is at least one interpretation in which all axioms of  $\Xi$  are true (i. e. a model of/for  $\Xi$  exists),  $\Xi$  is consistent. Moreover, as all axioms of  $\Xi$  are true in both "absolutely opposite" interpretations, namely, the rationalism-a-priori-ism and the sensualism-empiricism ones, the two "opposites" are synthesized by  $\Xi$  without a logic contradiction.

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## **MODELS FOR THE FORMAL AXIOMATIC EPISTEMOLOGY THEORY $\Xi$**

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### Annotation

The formal axiomatic theory in question is defined, and the problem of its logic consistency is investigated. For the first time such significantly different interpretations of the axiom system  $\Xi$  are submitted which are models of/for  $\Xi$ . By means of these models it is demonstrated that the theory in question is consistent.

Key concepts:

formal-axiomatic-theory; epistemology; interpretation; model; consistency.