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Логика знания и алгебра формальной аксиологии: некая формальная аксиоматическая теория эпистемологии Сигма, используемая для точного определения того экзотического условия, при котором доктрина Юма и Мура о логически непреодолимой пропасти между суждениями о бытии и суждениями о ценности фальсифицируется

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Впервые в формальной аксиоматической теории знания Сигма формально доказана теорема, означающая (в предложенной точно определенной интерпретации), что, при условии априорности знания, утверждение формально-аксиологической эквивалентности моральных ценностных функций логически эквивалентно утверждению логической эквивалентности соответствующих утверждений о бытии. Впервые продемонстрировано, что эта теорема отрицает универсальность упомянутой концепции Юма и Мура. Дано точное определение упомянутой формальной аксиоматической теории Сигма, являющейся результатом логической формализации универсальной философской эпистемологии; а также предложена некая релевантная интерпретация этой формальной теории. Представленное формальное доказательство вышеупомянутой теоремы может быть проверено читателями шаг за шагом, так как оно построено в соответствии со стандартами формализма (в обосновании математики).

Ключевые слова: формальная-аксиоматическая-теория-знания, знание-a-priori, двузначная-алгебра-формальной-аксиологии, формально-аксиологическая-эквивалентность, моральная-ценностная-функция, гильотина-Юма, натуралистические-ошибки-в-этике, дуализм-факта-и-ценности.

> Although the dogma of a fact-value dichotomy has long held sway in scientific and philosophical discourse and practice, it has been called into question by some of our most eminent thinkers: to name but a few, Charles S. Peirce, William James, F. C. S. Schiller, John Dewey, C. I. Lewis, Iris Murdoch, Philippa Foot, Donald Davidson, Hilary Putnam, Ruth Anna Putnam, Richard Rorty, and John McDowell, all of whom helped show how the assumed heterogeneity of descriptive and evaluative considerations and judgments seriously misrepresents the nature and operation of both.
>
> (Giancarlo Marchetti and Sarin Marchetti 2017)

1. Introduction: the dichotomy between statements of being and statements of value. According to Adler (Adler 1981), the strict separation of facts and values is one of the six great philosophical ideas created by the Western-world philosophy. According to Putnam (Putnam 1981: 127-128), unfortunately, in contemporary philosophy the absolute (unconditioned) dichotomy of facts and values has become a "cultural *institution*". Initially this *institution* (the social norm of/for philosophizing of being and value) was established by "Principia Ethica" (Moore 1903) which was a manifest proclamation of the institution in question. Although some important prerequisites for establishing the institution of logically unbridgeable gap between corresponding statements of being and statements of value existed in previous times, especially in writings by Hume (Hume 1874; 1994; 1998),

the absolute separation as a perfectly universal unconditioned principle was formulated manifestly by Moore. Since the beginning of XX century to nowadays, plenty of respectable philosophers have elaborated his conception with enthusiasm and have developed it consistently to its logical end. However, there were also eminent opponents of the dualism in history of philosophy, for example, Dewey (Dewey 1903; 1929; 1938; 1939). He was not the only criticizer of fact/value dichotomy. Another well-known resolute criticizer of the dichotomy was Putnam who believed that the extremist-minded adherents of absolute separation between statements of being and statements of value had arrived to an absurdity which contained a danger for philosophy and methodology of science (Putnam 1981: 127-1149, 201-217). Putnam has argued that there is an essential linkage (a necessary relationship) between facts and values: the fact/value dichotomy has collapsed (Putnam 1981; 2002; 2004; 2015; 2017). Hence, conjunction of relevant Moore's and Putnam's statements looks like a logical contradiction. This makes the hard problem to be discussed, formalized, and investigated by means of symbolic logic machinery with a view for an acceptable solution below in the present article. Thus, I accept the invitation to attempt somehow to overcome the dichotomy dogma which (invitation) has been proclaimed in the preface to the very interesting book edited by G. Marchetti and S. Marchetti (Marchetti G., Marchetti S. 2017).

The literature relevant to the theme is huge; for instance, writings by Adler (Adler 1981); Ayer (Ayer 1952; 1954); Lewis (Lewis 1946); MacIntyre (MacIntyre 1981); Mackie (Mackie 1946; 1977); G. Marchetti and S. Marchetti (Marchetti G., Marchetti S. 2017); Russell (Russell 1914; 1918; 1940; 1948; 1997); Schiller (Schiller 1903); Stevenson (Stevenson 1937; 1944; 1963); Weber (Weber 2017), etc., not homogeneous, and sometimes even contradictory. Therefore, for the sake of convenience and simplicity, let us divide the huge material into three categories. Let the first category of writings be composed by the works of adherents of the absolute separation between statements of being and statements of value as a *general principle possessing no limitations*, i.e. as a *not-falsifiable universal dualism* statement. Let us call authors of these writings the *absolute (unconditional) separatists*. In my opinion, the first category may be exemplified by Adler (Adler 1981), Ayer (Ayer 1952; 1954), Carnap (Carnap 1931; 1956), Moore (Moore 1903), Russell (Russell 1914; 1918; 1940; 1948; 1997), Schlick (Schlick 1974; 1978; 1979), Stevenson (Stevenson 1937; 1944; 1963), and Wittgenstein (Wittgenstein 1992). The second category of writings is composed by the works of those authors whose relevant statements are *contrary* to the first category ones. Let us call them the *absolute anti-separatists* insisting upon existence of necessary relationship (essential entanglement) or deep intertwinement between corresponding statements of being and statements of value as a *general principle possessing no limitations*, i.e. as a *not-falsifiable universal anti-dualism* principle. In my opinion, the second category may be exemplified by although not reduced to Dewey (Dewey 1903; 1929; 1938; 1939), Morris (Morris 1963), Schiller (Schiller 1903), and Putnam (Putnam

1981; 2002; 2004; 2015; 2017). I think that the above-mentioned two allegedly not-falsifiable universal principles (the *absolute* dualism and the *absolute* anti-dualism) are in *contrariety* relation to each other: they cannot be both true, but they can be both false. Therefore, now it is worth attracting attention to the third option of logical relationship between corresponding statements of being and statements of value, namely, the option of both: (A) there is a *limited* logic-unity (*conditioned* logic-equivalence) between corresponding statements of being and statements of value; (B) there is a *limited* logic-separation (*conditioned* logic-non-equivalence) between them. Affirming the conjunction of (A) and (B) means that both the dualism and the anti-dualism are not absolutely universal but falsifiable; they are *relatively (conditionally)* universal. Domains of validity of both the dualism and the anti-dualism are not completely infinite but quite definite. Moreover, in principle, it is possible precisely to define *the exotic (rare)* concrete *condition* under which (A) takes place, but (B) does not take place. Also, in principle, it is possible precisely to define the *usual (habitual)* concrete *condition* under which (B) takes place, but (A) does not take place. The above-indicated third option of logical relationship between corresponding statements of being and statements of value is to be investigated in the present paper.

Concerning the contemporary literature on the theme, I would like to make the following critical remarks. Being formulated *in general*, the nontrivial fundamental problem under investigation is the one of existence of logical connection between corresponding statements of *being* and statements of *value*. However, there is a strong tendency completely to *reduce* the *general* formulation of the problem to its *special particular* case, namely, to the problem of existence of logical connection between corresponding *facts* and *values* (evaluations). Thus, meanings of "fact" and "being" are identified completely (are used as synonyms). I think that, generally speaking, such identifying "fact" and "being" is not valid. In this paper I shall abstain from such identifying. In history of philosophy there is an intellectually respectable tradition to define *facts* as *contingent* truths or *contingent* events. This tradition may be exemplified by Leibniz (Leibniz 1903; 1952; 1969; 1971; 1981) and Carnap (Carnap 1931; 1956). I shall follow this tradition in the present paper. Also, I shall follow the negative attitude to the two dogmas of empiricism which (attitude) has been developed by Quine (Quine 1980).

Another critical remark: Being formulated *in general*, the above-raised nontrivial fundamental problem deals with *values* (or statements of *value*). Values are either absolute or relative. Statements of positive value (goodness) are either *contingently* positive or *necessarily* positive (*absolute* goodness). Is there an *absolute (necessary)* good? As a rule, logical positivists resolutely answered this question negatively: Russel (Russel 1914; 1918; 1940; 1948; 1997) and Mackie (Mackie 1946; 1977) are representative examples. Thus, in the philosophical empiricism, meanings of "value" and "relative (contingent) value" are identified completely (are used as synonyms) either subconsciously or deliberately. I think that generally speaking, such iden-

tifying "value" and "relative (contingent) value" is not valid. In this paper I shall abstain from such identifying.

However, it is the fact that within the *empirical* philosophy of values, which (philosophy) dominates in the contemporary literature on the theme, evaluations are considered as statements of *contingent (relative)* values. If the *general* formulation of the problem is completely reduced to the *particular* case, then, I think that Hume, Russell, Moore, and Wittgenstein are right: there are no necessary logical relations between facts (=*contingently* true statements of *contingent* events) and *contingently* true statements of *contingent* (not absolute but *relative*) values of these events.

However, in my opinion, the mentioned complete reduction of the problem to its particular case is not acceptable from the perfect theoretical philosophy viewpoint because there is another also very important special particular case of the problem, namely, the nontrivial problem of existence of necessary logical connection between corresponding statements of *necessary* being and statements of *necessarily* positive value (*absolute* goodness). *Empiricist*-minded philosophers have *ignored* this very important problem *on principle*. I believe that such ignoring is a defective (one-sided) attitude. The relevant views of *rationalist*-minded philosophers dealing with *a-priori* knowledge, for instance, the relevant *rationalistic* ideas by Descartes (Descartes 1994a; 1994b) and Leibniz (Leibniz 1903; 1952; 1969; 1971; 1981) must be taken into an account. The philosophical theology, especially the *theodicy* by Leibniz (Leibniz 1952), is also quite relevant to the case as it deals with *necessary logical connecting the necessary being with the necessary goodness*.

To eliminate the above-indicated defect, below in this article I am to develop a *synthetic (two-sided)* attitude to the problem by using such a formal axiomatic epistemology theory Σ, which *unites* consistently the *contrary* paradigms of empiricism and a-priori-ism. As the problem is very difficult, I shall divide it into two parts and attack the parts separately one after another. This is an effective strategy deliberately used by politicians, military-men and mathematicians.

First of all, I am to criticize Moore's idea of absolutely universal (unconditional) logical dichotomy between corresponding statements of being and statements of value. I am to do this by deliberate inventing (intentional constructing) a counter-example falsifying Moore's conception. Such criticizing Moore's allegedly universal doctrine of naturalistic fallacies in ethics is a significant novelty: the *deductive* logic apparatus exploited for falsifying the doctrine of Moore in this article differs much from the *inductive* logic methods used by the overwhelming majority of his opponents hitherto. I mean systematical exploiting (1) two-valued algebra of formal ethics as formal axiology (Lobovikov 1980; 1984; 1988; 1999; 2014; 2018a; 2019) and (2) a formal axiomatic epistemology theory Σ (Sigma) to be precisely defined below in the present article. Hereafter the terms "proof" and "theorem" are used in those special meanings which have been defined precisely in XX century mathematical logic by the formalists (Weir 2019). Namely, by

definition, a proof of a formula as a theorem in an axiomatic theory is such a finite succession of formulae of the theory, in which succession: 1) the theorem is the last formula of the succession; 2) any formula belonging to the succession is either an axiom of the theory; or a formula obtained from previous formulae of the succession by an inference-rule of the theory. Originally, an attempt to criticize Moore's doctrine of the naturalistic fallacies in ethics by exploiting formal-deductive-inference construction in a formal axiomatic epistemology theory was undertaken by Lobovikov (Lobovikov 2017). However, the attempt was accomplished within not Σ but another formal axiomatic epistemology theory, and the theorem formally proved by Lobovikov (Lobovikov 2017) differs significantly from the one formally proved in the present article.

Within the formal axiomatic epistemology theory Σ, below a *formal deductive proof of formula-scheme* ($A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$) *as a theorem-scheme (in Σ)* is constructed for the first time. According to that semantics of the artificial language of formal theory Σ, which (semantics) is defined precisely below in this article, the theorem-scheme ($A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$) represents (in Σ) the above promised precise definition of the exotic condition under which the theory of Moore is falsified.

According to the below-given interpretation of Σ, the formula $A\alpha$ represents the *assumption of a-priori-ness of knowledge*. In the interpretation under discussion, "$\supset$" is "classical (material) implication". Formally to prove that ($A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$) is a theorem-scheme in Σ, and attentively to examine the formal proof, it is indispensable to have exact definitions of the terms involved into the discourse. Therefore, let us start with submitting precise definitions of the notions relevant to the case.

2. Precisely Defining the Formal Axiomatic Epistemology Theory Σ. The present paragraph of this paper is targeted at making the reader acquainted with the formal axiomatic epistemology theory Σ which is a result of developing further (complementing substantially) the axiomatic epistemology system Ξ originally submitted by Lobovikov (Lobovikov 2018b).

According to the definition, the logically formalized axiomatic epistemology system Ξ contains all symbols (of the alphabet), expressions, formulae, axioms, and inference-rules of the formal axiomatic epistemology theory Ξ (Lobovikov 2018b) which is based on the classical propositional logic. But in Σ several significant aspects are added to the formal theory Ξ. In result of these additions the alphabet of Σ's object-language is defined as follows:

> 1) Small Latin letters q, p, d (and the same letters possessing lower number indexes) are symbols belonging to the alphabet of object-language of Σ; they are called "*propositional* letters". *Not all small Latin letters are propositional ones* in the alphabet of Σ's object-language, as, by this definition, small Latin letters belonging to the set {g, b, e, n, x, y, z, t} are excluded from the set of *propositional* letters.

2) Logic symbols $\neg$, $\supset$, $\leftrightarrow$, &, $\vee$ called "classical negation", "material implication", "equivalence", "conjunction", "not-excluding disjunction", respectively, are symbols belonging to Σ's object-language alphabet.

3) Elements of the set of modality-symbols {□, K, A, E, S, T, F, P, Z, G, W, O, B, U, Y} belong to Σ's object-language alphabet.

4) Technical symbols "(" and ")" ("round brackets") belong to Σ's object-language alphabet. The round brackets are exploited in this paper as usually in symbolic logic.

5) Small Latin letters x, y, z (and the same letters possessing lower number indexes) are symbols belonging to Σ's object-language-alphabet (they are called "axiological variables").

6) Small Latin letters "g" and "b" called *axiological constants* belong to the alphabet of object-language of Σ.

7) The following not-indexed capital Latin letters – L, V, I, D, J, N, and the capital Latin letters possessing number indexes – K^2, W^2, A^2, A_k^n, B_i^n, C_j^n, D_m^n, … belong to the object-language-alphabet of Σ (they are called "*axiological-value-functional symbols*"). The upper number index *n* informs that the indexed symbol is *n*-placed one. Nonbeing of the upper number index informs that the symbol is determined by one axiological variable. The value-functional symbols may have no lower number index. If lower number indexes are different, then the indexed functional symbols are different ones.

8) Symbols "[" and "]" ("square brackets") also belong to the object-language-alphabet of Σ, but in this theory they are exploited in an very *unusual* way. Although, from the psychological viewpoint, square brackets and round ones look approximately identical and are used very often as synonyms, in the present article they have *qualitatively different* meanings (roles): exploiting round brackets is purely technical as usually in symbolic logic; square-bracketing has an *ontological* meaning which is to be defined below while dealing with *semantic* aspect of Σ. Moreover, even at syntax level of Σ's object-language, being not purely technical symbols, square brackets *play a very important role* in the below-given definition of the general notion "formula of Σ" and in the below-given formulations of some axiom-schemes of Σ.

9) An unusual artificial symbol "=+=" called "*formal-axiological equivalence*" belongs to the alphabet of object-language of Σ. The symbol "=+=" also *plays a very important role* in the below-given definition of the general notion "formula of Σ" and in the below-given formulations of some axiom-schemes of Σ.

10) A symbol belongs to the alphabet of object-language of Σ, if and only if this is so owing to the above-given items 1) – 9) of the present definition.

A finite succession of symbols is called an *expression* in the object-language of Σ, if and only if this succession contains such and only such sym-

bols which belong to the above-defined alphabet of Σ's object-language.

Now let us define precisely the general notion "*term* of Σ":

1) the *axiological variables* (from the above-defined alphabet) are terms of Σ;

2) the *axiological constants* belonging to the alphabet of Σ, are terms of Σ;

3) If Φ_k^n is an *n-placed axiological-value-functional symbol* from the above-defined alphabet of Σ, and t_i, ... t_n are *terms* (of Σ), then $\Phi_k^n t_i$, ... tn is a term (compound one) of Σ (here it is worth remarking that symbols t_i, ... t_n belong to the meta-language, as they stand for *any* terms of Σ; the analogous remark may be made in relation to the symbol Φ_k^n which also belongs to the meta-language);

4) An expression in object-language of Σ is a term of Σ, if and only if this is so owing to the above-given items 1) – 3) of the present definition.

Now let us make an agreement that in the present paper, small Greek letters α, β, and γ (belonging to meta-language) stand for *any* formulae of Σ. By means of this agreement the general notion "*formulae* of Σ" is defined precisely as follows.

1) All the above-mentioned propositional letters are formulae of Σ.

2) If α and β are formulae of Σ, then all such expressions of the object-language of Σ, which possess logic forms $\neg\alpha$, $(\alpha \supset \beta)$, $(\alpha \leftrightarrow \beta)$, $(\alpha \,\&\, \beta)$, $(\alpha \vee \beta)$, are formulae of Σ as well.

3) If t_i and t_k are terms of Σ, then $(t_i{=}{+}{=}t_k)$ is a formula of Σ.

4) If t_i is a term of Σ, then $[t_i]$ is a formula of Σ.

5) If α is a formula of Σ, and meta-language-symbol Ψ stands for any element of the set of modality-symbols {□, K, A, E, S, T, F, P, Z, G, W, O, B, U, Y}, then any object-language-expression of Σ possessing the form Ψα, is a formula of Σ as well. (Here, the meta-language-expression Ψα is not a formula of Σ, but a scheme of formulae of Σ.)

6) Successions of symbols (belonging to the alphabet of the object-language of Σ) are formulae of Σ, if and only if this is so owing to the above-given items 1) – 5) of the present definition.

Now let us introduce the elements of the above-mentioned set of modality-symbols {□, K, A, E, S, T, F, P, Z, G, W, O, B, U, Y}. Symbol □ stands for the alethic modality "necessary". Symbols K, A, E, S, T, F, P, Z, respectively, stand for modalities "agent *Knows* that...", "agent *A-priori knows* that...", "agent *Empirically (a-posteriori)* knows that...", "under some conditions in some space-and-time a person (immediately or by means of some tools) *Sensually perceives* (has *Sensual verification*) that...", "it is *True* that...", "person has *Faith* (or believes) that...", "it is *Provable* that...", "there is *an algorithm* (a machine could be constructed) *for deciding* that...".

Symbols G, W, O, B, U, Y, respectively, stand for modalities "it is *(morally) Good* that...", "it is *(morally) Wicked* that...", "it is *Obligatory* that ...", "it

is *Beautiful* that …", "it is *Useful* that …", "it is *pleasant* that …". Meanings of the mentioned symbols are defined (indirectly) by the following schemes of own (proper) axioms of epistemology system Σ which axioms are added to the axioms of classical propositional logic. Schemes of axioms and inference-rules of the classical propositional logic are applicable to all formulae of Σ. More detailed introduction and content-discussion of the modalities can be found in (Lobovikov 2018b).

Axiom scheme AX-1: $A\alpha \supset (\Box\beta \supset \beta)$.
Axiom scheme AX-2: $A\alpha \supset (\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta))$.
Axiom scheme AX-3: $A\alpha \leftrightarrow (K\alpha \ \& \ (\Box\alpha \ \& \ \Box \neg S\alpha \ \& \ \Box(\beta \leftrightarrow \Omega\beta)))$.
Axiom scheme AX-4: $E\alpha \leftrightarrow (K\alpha \ \& \ (\neg \Box\alpha \vee \neg \Box \neg S\alpha \vee \neg \Box(\beta \leftrightarrow \Omega\beta)))$.
Axiom scheme AX-5: $(\Box\beta \ \& \ \Box\Box\beta) \supset \beta$.
Axiom scheme AX-6: $(t_i{=}{+}{=}t_k) \supset (G[t_i] \leftrightarrow G[t_k])$.
Axiom scheme AX-7: $(t_i{=}{+}{=}g) \supset \Box G[t_i]$.
Axiom scheme AX-8: $(t_i{=}{+}{=}b) \supset \Box W[t_i]$.
Axiom scheme AX-9: $(G\alpha \supset \neg W\alpha)$.
Axiom scheme AX-10: $(W\alpha \supset \neg G\alpha)$.

In AX-3 and AX-4, the symbol Ω (belonging to the meta-language) stands for any element of the set $\mathfrak{R}$ = {□, K, T, F, P, Z, G, O, B, U, Y}. Let elements of $\mathfrak{R}$ be called "*perfection*-modalities" or simply "perfections".

The axiom-schemes AX-9 and AX-10 are not new in evaluation logic: one can find them in the famous monograph by (Ivin 1970). But the axiom-schemes AX-6, AX-7, AX-8 are new ones: they have not been published hitherto.

3. Defining Semantics of/for ∑. Meanings of the symbols belonging to the alphabet of object-language of Σ owing to the items 1—3 of the above-given definition of the alphabet are defined by the classical propositional logic.

For defining semantics of specific aspects of object-language of formal theory Σ it is necessary to define a set Δ (called "field of interpretation") and an interpreter called "valuator (evaluator)" Θ.

In a standard *interpretation* of formal theory Σ, the set Δ (field of interpretation) is such a set, every element of which has: (1) one and only one *axiological* value from the set {good, bad}; (2) one and only one *ontological* value from the set {exists, not-exists}.

The *axiological variables* x, y, z range over (take their values from) the set Δ.

The *axiological constants* "g" and "b" mean, respectively, "good" and "bad".

It is presumed here that *axiological evaluating* an element from the set Δ, i.e. ascribing to this element an *axiological value* from the set {good, bad} is performed by a quite definite (perfectly fixed) individual or collective valuator (evaluator) Θ. It is obvious that changing Θ can result in changing valuations of elements of Δ. But *laws of two-valued algebra of formal axiology* do not depend upon changes of Θ as, by definition, formal-axiological laws

of this algebra are such and only such *constant evaluation-functions which obtain the value "good"* independently from any changes of valuators. Thus, generally speaking, Θ is a *variable* which takes its values from the set of all possible evaluators (individual or collective – it does not matter). Nevertheless, a *concrete interpretation* of formal theory Σ is *necessarily fixing* the value of Θ; changing the value of the variable Θ is changing the concrete interpretation.

In a standard *interpretation* of formal theory Σ, *ontological constants* "e" and "n" mean, respectively, "exists" and "not-exists". Thus, in a standard interpretation of formal theory Σ, one and only one element of the set {{g, e}, {g, n}, {b, e}, {b, n}} corresponds to every element of the set Δ. The *ontological constants* "e" and "n" belong to the meta-language. (According to the above-given definition of Σ's object-language-alphabet, "e" and "n" do not belong to the object-language.) But the *ontological constants are indirectly represented at the level of object-language by square-bracketing*: "t_i exists" is represented by $[t_i]$; "t_i not-exists" is represented by $\neg[t_i]$. Thus square-bracketing is a very important aspect of the system under investigation.

N-placed terms of Σ are interpreted as *n-ary algebraic operations (n-placed evaluation-functions)* defined on the set Δ.

Speaking of *evaluation-functions* means speaking of the following mappings (in the proper mathematical meaning of the word "mapping"): {g, b} → {g, b}, if one speaks of the evaluation-functions determined by *one* evaluation-variable; {g, b}×{g, b} → {g, b}, where "×" stands for the Cartesian product of sets, if one speaks of the evaluation-functions determined by two evaluation-variables; $\{g, b\}^N \rightarrow \{g, b\}$, if one speaks of the evaluation-functions determined by N evaluation-variables, where N is a finite positive integer. For instantiating the psychologically not-habitual general notion "*evaluation-function*" systematically used in two-valued algebra of ethics as formal axiology, evaluation-functions *Lx, Vx, Qx, Dx, Jx, Nx* are defined precisely by the below evaluation-table 1. (For correct understanding contents of this paper, it is worth emphasizing here that in the semantics of Σ, the above-used symbols *Lx, Vx, Ix, Dx, Jx, Nx* mean *not predicates but terms*. Being given a relevant interpretation, the expressions $(t_i{=}{+}{=}t_k)$, $(t_i{=}{+}{=}g)$, $(t_i{=}{+}{=}b)$ are representations of *predicates* in Σ.)

Table 1. One-placed evaluation-functions

x	*Lx*	*Vx*	*Ix*	*Dx*	*Jx*	*Nx*
g	g	b	b	g	g	b
b	b	g	b	g	b	g

The one-placed term *Lx* is interpreted in this article as *one-placed evaluation-function "freedom for* (what, whom) *x*". The one-placed term *Vx* is interpreted in this article as one-placed evaluation-function "freedom from (what, whom) x". In the interpretation submitted in this paper, the

term *Ix* stands for the evaluation-function ""*absolute freedom for arbitrary (contingent) choice between* being of *x* and nonbeing of *x*", i.e. "*freedom for (tolerance to) both: realizing* (what, whom) *x* and *not-realizing* (what, whom) *x*". In other words, *Ix* means act of uniting "*freedom from* (what, whom) *x*" and "*freedom from* nonbeing of (what, whom) *x*". Thus, in this paper, the term *Ix* is interpreted as a *negative*-constant-evaluation-function, i.e. as a f*ormal-axiological contradiction*. In the interpretation given in this paper, the term *Dx* stands for the evaluation-function "*x's being free from Ix, i.e. x's being free from (somebody's) arbitrary treating* (what, whom) *x*". Thus, in natural language, humans have not one and the only, and even not two, but four mathematically different meanings of the word "freedom". (Unfortunately, even today philosophers and linguists do not recognize the very important mathematical difference among the four.) Finally, in the above table 1; the symbol *Jx* stands for the evaluation-function "*being (existence) of* (what, whom) *x*"; *Nx* stands for the evaluation-function "*non-being (non-existence) of* (what, whom) *x*".

For instantiating the general notion "*two-placed evaluation-function*" also systematically exploited in two-valued algebra of ethics as formal axiology, one can use (Lobovikov 1980; 1984; 1988; 1999).

If t_i is a term of Σ, then, being interpreted, formula $[t_i]$ of Σ is an *either true or false proposition* "t_i exists". In a standard interpretation, formula $[t_i]$ is true if and only if t_i has the *ontological value* "e (exists)" in that interpretation. The formula $[t_i]$ is a false proposition in a standard interpretation, if and only if t_i has the *ontological value* "n (not-exists)" in that interpretation.

Given a relevant interpretation, the formula $(t_i=+=t_k)$ of Σ is translated into natural language by the proposition "t_i is f*ormally-axiologically equivalent* to t_k", which proposition is true if and only if (in the interpretation) the terms t_i and t_k have identical *axiological values* from the set {good, bad} under any possible combination of *axiological values* of their *axiological variables*.

Now, having introduced and defined precisely the substantially new notions essentially involved into the discourse, let us move directly to the above-promised formal proof construction.

4. Formal proof of the theorem scheme $(A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k])))$ in Σ. The proof of $(A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k])))$ in Σ is the following succession of formulae schemes.

1. $A\alpha \leftrightarrow (K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta)))$ by axiom-scheme AX-3.

2. $A\alpha \leftrightarrow (K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box([t_i] \leftrightarrow G[t_i])))$ from 1 by substituting: G for Ω; $[t_i]$ for β.

3. $A\alpha \supset (K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box([t_i] \leftrightarrow G[t_i])))$ from 2 by the rule of ↔ elimination.

4. $A\alpha$ assumption.

5. $K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box([t_i] \leftrightarrow G[t_i]))$ from 3 and 4 by *modus ponens*.

6. $\Box([t_i] \leftrightarrow G[t_i])$ from 5 by the rule of eliminating &.

7. $([t_i] \leftrightarrow G[t_i])$ from 4 and 6 by a rule of □ elimination. (The □ elimination rule is *derivative* one[1].)

8. $A\alpha \leftrightarrow (K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box([t_k] \leftrightarrow G[t_k])))$ from 1 by substituting: G for Ω; $[t_k]$ for β.

9. $A\alpha \supset (K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box([t_k] \leftrightarrow G[t_k])))$ from 8 by the rule of eliminating ↔.

10. $K\alpha \,\&\, (\Box\alpha \,\&\, \Box\neg S\alpha \,\&\, \Box([t_k] \leftrightarrow G[tk]))$ from 4 and 9 by *modus ponens*.

11. $\Box([t_k] \leftrightarrow G[t_k])$ from 10 by the rule of eliminating &.

12. $([t_k] \leftrightarrow G[t_k])$ from 4 and 11 by the rule of □ elimination.

13. $(t_i{=}{+}{=}t_k) \leftrightarrow (G[t_i] \leftrightarrow G[t_k])$ axiom-scheme AX-6.

14. $(t_i{=}{+}{=}t_k) \supset (G[t_i] \leftrightarrow G[t_k])$ from 13 by the rule of ↔ elimination.

15. $(t_i{=}{+}{=}t_k)$ assumption.

16. $(G[t_i] \leftrightarrow G[t_k])$ from 14 and 15 by *modus ponens*.

17. $([t_i] \leftrightarrow G[t_k])$ from 7 and 16 by the rule of transitivity of ↔.

18. $(G[t_k] \leftrightarrow [t_k])$ from 12 by the rule of commutativity of ↔.

19. $([t_i] \leftrightarrow [t_k])$ from 17 and 18 by the rule of transitivity of ↔.

20. $A\alpha, (t_i{=}{+}{=}t_k) \vdash ([t_i] \leftrightarrow [t_k])$ by the succession 1—19.

21. $A\alpha \vdash (t_i{=}{+}{=}t_k) \supset ([t_i] \leftrightarrow [t_k])$ from 20 by the rule of ⊃ introduction.

22. $(G[t_i] \leftrightarrow G[t_k]) \supset (t_i{=}{+}{=}t_k)$ from 13 by the rule of ↔ elimination.

23. $([t_i] \leftrightarrow [t_k])$ assumption.

24. $(G[t_i] \leftrightarrow [t_i])$ from 7 by the rule of commutativity of ↔.

25. $(G[t_i] \leftrightarrow G[t_k])$ from 24 and 17 by the rule of transitivity of ↔.

26. $(t_i{=}{+}{=}t_k)$ from 22 and 25 by modus ponens.

27. $A\alpha, ([t_i] \leftrightarrow [t_k]) \vdash (t_i{=}{+}{=}t_k)$ by the succession 1—26.

28. $A\alpha \vdash ([t_i] \leftrightarrow [t_k]) \supset (t_i{=}{+}{=}t_k)$ from 27 by the rule of ⊃ introduction.

29. $A\alpha \vdash ((t_i{=}{+}{=}t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$ from 28 and 21 by the rule of ↔ introduction.

30. $\vdash A\alpha \supset ((t_i{=}{+}{=}t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$ from 28 by the rule of ⊃ introduction.

Here you are[2].

5. Discussing the Theorem Scheme and Arriving to the Conclusion. Moore (Moore 1903) undertook an attempt of systematical critique of arguments demonstrating rational moral philosophy statement of the existence of logical relationship between corresponding statements of being and statements of moral goodness (positive moral value). All possible attempts logically to bridge the gap between the two qualitatively different species of statements were labeled by him "naturalistic fallacies in ethics".

[1] It is formulated as follows: $A\alpha, \Box\beta \vdash \beta$. This rule is not included into the above-given definition of Σ, but it is easily *derivable* in Σ by means of the axiom scheme AX-1 and *modus ponens*. (The rule $\Box\beta \vdash \beta$ is not derivable in Σ, and also Gödel's necessitation rule is not derivable in Σ. Nevertheless, a limited or conditioned necessitation rule is derivable in Σ, namely, $A\alpha, \Box\beta \vdash \beta$.)

[2] I am grateful to Grigori Olkhovikov for his examining the proof and for suggesting an option of making it more short one.

However, in relation to Moore's *unconditional* negating the arguments *in general*, the theorem ($A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$) formally proved above in the present article (within the theory Σ) is a counter-example; under the *rare (exotic)* condition of knowledge *a-priori-ness*, formal-logic deriving statements of positive moral value from corresponding statements of being can be valid. In the above-defined interpretation of Σ: ($t_i=+=t_k$) is a *purely evaluative* statement as it is a formal-axiological equivalence of evaluation-functions; ($[t_i] \leftrightarrow [t_k]$) is a *purely ontological* statement as it is a logical equivalence of corresponding statements of being, consequently, the *purely ontological* statement implies logically the corresponding *purely evaluative* one (and conversely) under the condition that $A\alpha$.

It is worth noting here that there are publications on metaphysics as formal axiology, for instance, by Lobovikov (Lobovikov 2007; 2015), in which (for the sake of avoiding paradoxes) the following formal rule is suggested: (A) it is forbidden logically to infer ($[t_i] \leftrightarrow [t_k]$) from ($t_i=+=t_k$), and (B) it is forbidden logically to infer ($t_i=+=t_k$) from ($[t_i] \leftrightarrow [t_k]$). This prohibition based on philosophical *empiricism* conceptions by Hume (Hume 1874; 1994; 1998) and Moore (Moore 1903) *seems* to be in logic contradiction with the above-proved theorem, but, in my opinion, it *only seems* so, as the domain of relevant applicability of the above-formulated prohibition has limits: the ban is *not absolutely, but relatively* valid.

The *empirical* moral philosophy doctrines by Hume and Moore are *only partly* adequate: they are universally true within the *restricted* domain of *facts (=contingent truths)* and *empirical* arguments; they are quite right *under the condition* that $E\alpha$. But *contingent moral truths* and *empirical* moral arguments were beyond the aim and subject-matter of the present paper; hence, they were not involved into the discourse intentionally. In the article an abstraction from empirical aspect of the problem under discussion is accepted (this explains using not all axiom schemes of Σ in this paper), consequently, the philosophical significance of the theorem formally proved in this paper is limited. Nevertheless, the unusual formal axiomatic theory Σ and the psychologically unexpected deductive inference of the *purely evaluative* statement from the corresponding *purely ontological* one (under the condition that $A\alpha$) is interesting theoretically and worth discussing.

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Knowledge logic and algebra of formal axiology: a formal axiomatic epistemology

theory Sigma used for precise defining the exotic condition under which Hume-and-Moore doctrine of logically unbridgeable gap between statements of being and statements of value is falsified

Abstract. For the first time, in the formal axiomatic epistemology theory Sigma such a theorem is formally proved which means (in the precisely defined interpretation) that under the condition of knowledge a-priori-ness, a statement of formal-axiological equivalence of moral-evaluation-functions is logically equivalent to logic equivalence of corresponding statements of being. For the first time it is shown that this theorem undermines universality of the conception of Hume and Moore. A precise definition is given for the formal axiomatic theory Sigma, which is a result of logical formalization of the universal philosophical epistemology; and a relevant interpretation of this formal theory is submitted. The formal proof of the theorem can be examined by readers step by step as it is accomplished in accordance with the formalism standards.

Keywords: formal-axiomatic-epistemology-theory; a-priori-knowledge; two-valued-algebra-of-formal-axiology; formal-axiological-equivalence; moral-evaluation-function; Hume-guillotine; naturalistic-fallacies-in-ethics; fact-value-dualism.

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